Achievable Rates for Network Coding on the Exchange Channel

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Overview

1. Introduction to Physical-Layer Network Coding (PNC)
   - PNC: Zhang/Liew/Lam (MobiCom, 2006)
   - AM/FW: Katti/Gollakota/Katabi (2007)
2. THP Physical-Layer Network Coding
3. Optimization and Comparisons
4. Improving the THP-based Scheme
5. Conclusions

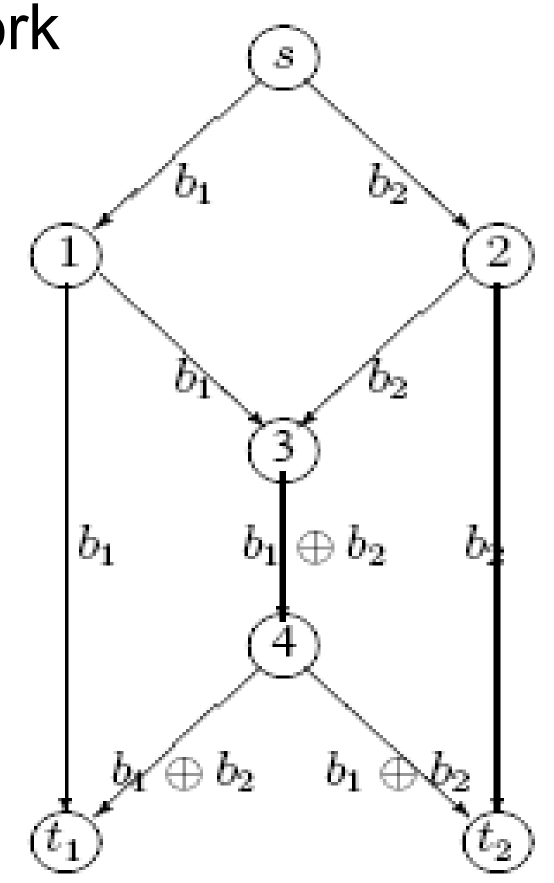
Network Coding

Idea: Packets are coded within the network rather than only at the edges.

Example: Try to send two bits from s to each of t_1 and t_2 in this network:

- routing and replication: bottleneck vertical lines make this impossible

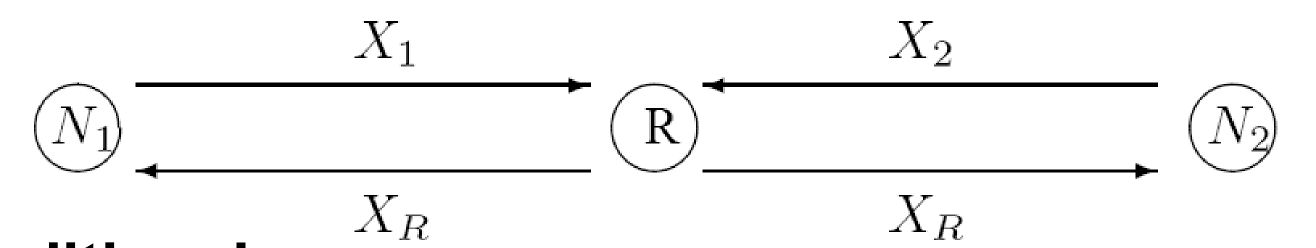
- with coding at intermediate nodes (as shown), goal can be achieved



[From Ahlswede et al (IEEE IT, 2000)]

The Exchange Channel

Goal: Nodes N_1 and N_2, which cannot hear each other, wish to exchange information D_1 and D_2 through an intermediate note R (the "relay").



Traditional:

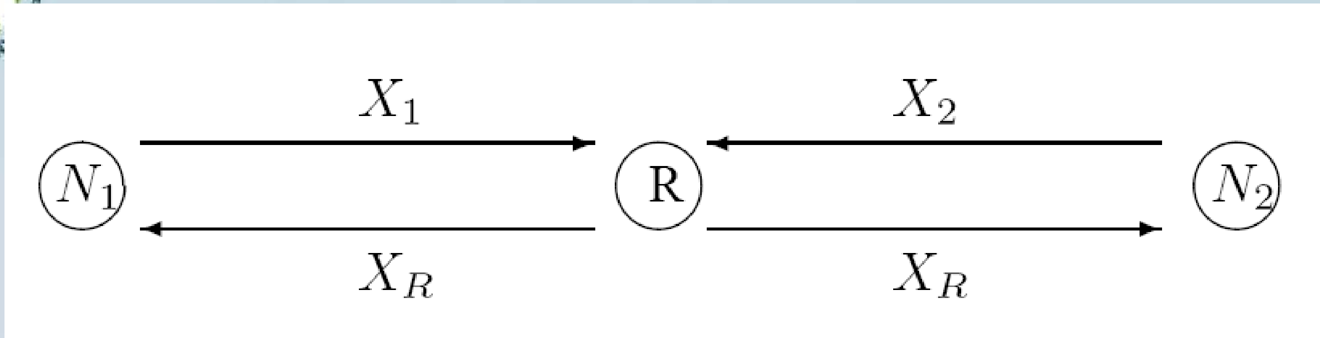
1. $N_1 \Longrightarrow R\ (D_1)$
2. $R \Longrightarrow N_2\ (D_1)$
3. $N_2 \Longrightarrow R\ (D_2)$
4. $R \Longrightarrow N_1\ (D_2)$

Network Coding:

1. $N_1 \Longrightarrow R\ (D_1)$
2. $N_2 \Longrightarrow R\ (D_2)$
3. $R \Longrightarrow N_1, N_2\ (D_1 \oplus D_2)$

Notes: N_1 has D_1, can decode D_2 from $D_1 \oplus D_2$ (analogous at N_2).

Physical Layer Network Coding – Zhang et al



1. D_1 and D_2 xmitted *simultaneously* on X_1 and X_2, respectively.
2. **<u>Key</u>**: The relay only decodes $D_1 \oplus D_2$.
3. Relay broadcasts $D_1 \oplus D_2$ during second time slot.

<u>Transmitters</u>

X_1 = -1 or +1 (corresponding to D_1)
X_2 = -1 or +1 (corresponding to D_2)

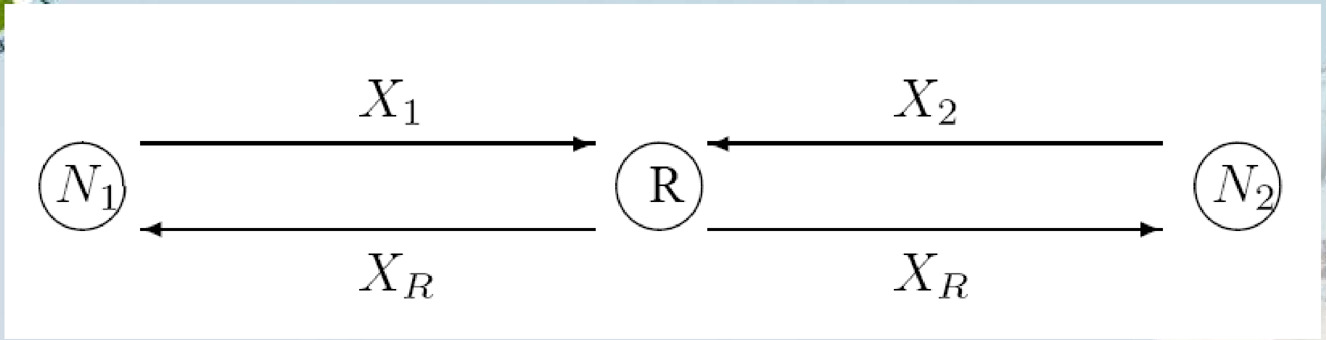
<u>Relay</u>

$X_1 + X_2 = 0 \Rightarrow D_1 \oplus D_2 = 1$

$X_1 + X_2 = \{-2, +2\} \Rightarrow D_1 \oplus D_2 = 0$

Advantage: Only 2 xmits (and P_e similar to receiving a single bit).
Disadvantage (major): (Complex) Channel gains must be pre-compensated for at X_1 and X_2.

Physical Layer Network Coding (Katti et al)



1. D_1 and D_2 xmitted *simultaneously* on X_1 and X_2, respectively.
2. The relay does not decode, but simply amplifies the received signal $X_1 + X_2$.
3. Relay broadcasts amplified $X_1 + X_2$ during second time slot.
4. **Key:** Clever noncoherent decoder at N_1 and N_2.

Advantage: Only 2 transmissions.

Disadvantage: Power inefficiency at relay. From N_2's perspective: half of the relay power is invested in a signal I already know.

Goals of this Paper

1. **Provide perspective on this emerging research area.**

The community is putting too much stock in "number of transmissions". We have long known that the multiple-access channel (MAC) followed by the broadcast channel (BC) can accomplish exchange in "2 transmissions".

2. **Provide a new PNC scheme to help remedy the shortcoming of AM/FW of Katti et al**

At N_1: $$\frac{1}{\sqrt{2+\sigma_R^2}}(X_1 + X_2 + W_R) + W_2$$

Focus on observation that relay is wasting half of its power from the perspective of either decoder. Any way to get that back?

Address Second Goal First

Basic Problem:

1. I put a number X in [-A, A] on a given channel.
2. You then put a number Y in [-A, A] on the same channel.
3. Each of us observe Z = X + Y in [-2A, 2A]. With knowledge of X, I easily can decode Y from Z.
   You can easily decode X from Z.

Question: Can I get away with a smaller range (i.e. lower power) than [-2A, 2A]?

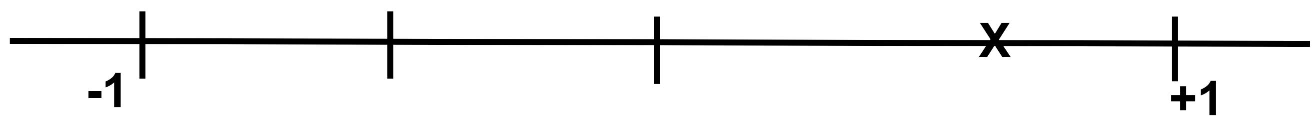
Answer: Reminiscent of “branch cut” from complex analysis. (also, Tomlinson-Harashima Precoding (THP)).

Idea of THP-based scheme

1. I put a number X in [-A, A] on a given channel.
2. You then put a number Y in [-A, A] on the channel.
3. Each of us observe Z = (X + Y) mod 2A in [-A, A].
4. With knowledge of X, I can decode Y. Likewise, you can decode X.

Example: from [-1,1] X = 0.7, Y = 0.8

- for Z = 0.7 + 0.8 = 1.5 mod 2 = -0.5.



The Schemes

Hence, we have the following schemes for comparison:

1. Multiple-access followed by broadcast (MAC/BC)

2. Coding at Network Level (Digital network coding - DNC)

3. Zhang/Liew/Lam – 2006 (Physical-layer network coding -PNC)

4. Katti/Gollakota/Katabi – 2007 (Amplify-forward – AM/FW)

5. THP-based PNC

Achievable Rates Analysis

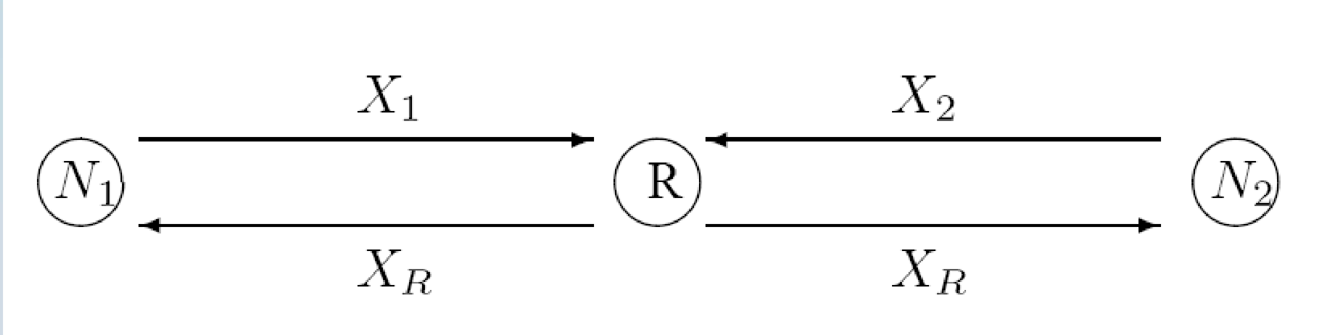
Assumptions:

- Exchange channel (additive white Gaussian noise – AWGN)
- Total power constraint (sources, relay)
- Symmetric (rate equal in each direction)
- Maximize symmetric rate

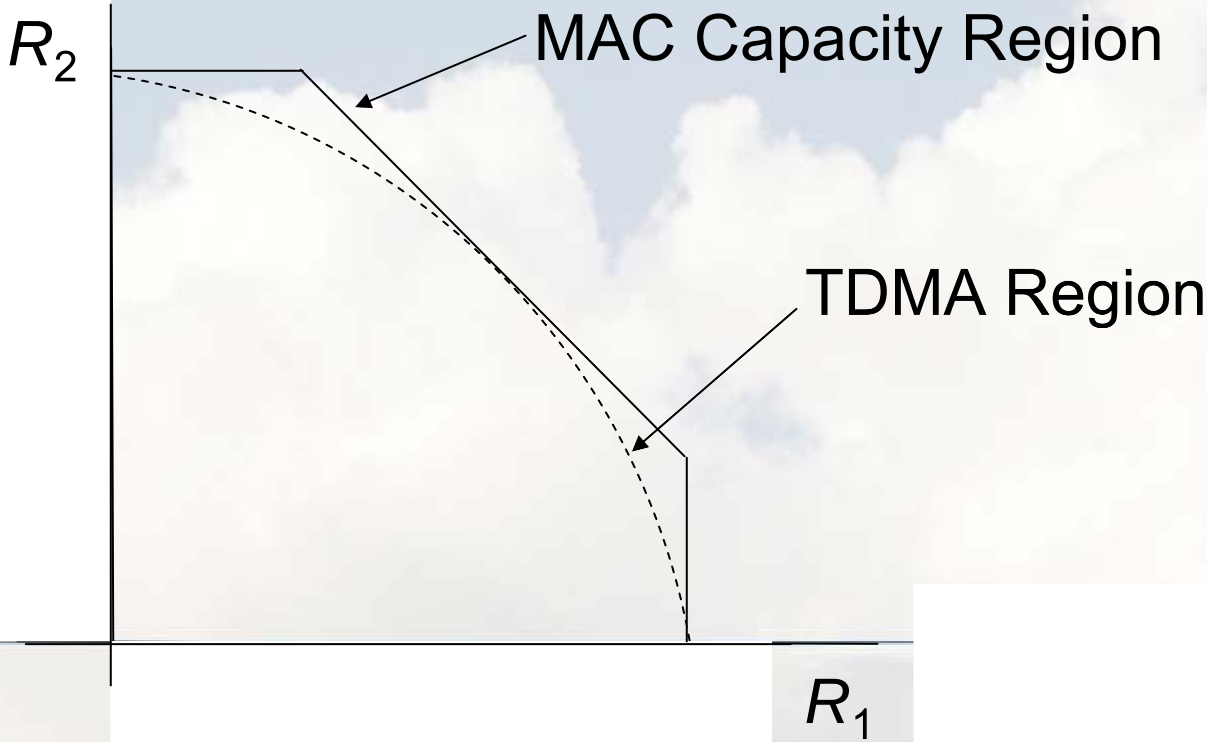
Method:

For each of the five schemes, optimize power and time allocation between source, relay transmissions (see paper for details of each).

Example: Optimization of MAC/BC



MAC Capacity Region:



Example: Optimization of MAC/BC II

P_1 = Power of a source.
P_R = Power of the relay.
ρ = % time sources xmit.

Sharing the sum rate.

Sum rate constraint dominates

From sources to relay: $$R_{N_1 \to R} = R_{N_2 \to R} \le \frac{1}{2}\left(\frac{1}{2}\log_2\left(1+\frac{2P_1}{\sigma^2}\right)\right)$$

$$= \frac{1}{4}\log_2\left(1+\frac{2P_1}{\sigma^2}\right)$$

Broadcast from relay: $$R_{R \to N_1} = R_{R \to N_2} \le \frac{1}{2}\log_2\left(1+\frac{P_R}{\sigma^2}\right)$$

Equal rate constraint: $$\frac{\rho}{4}\log_2\left(1+\frac{2P_1}{\sigma^2}\right) = \frac{(1-\rho)}{2}\log_2\left(1+\frac{P_R}{\sigma^2}\right)$$

Power constraint: $$2\rho P_1 + (1-\rho)P_R \le P$$

Solve for ρ and P_1.

THP-based PNC

Need to find the input distribution that maximizes the mutual information given A:

$$I(R_2; X_1) = h(R_2) - h(R_2 \mid X_1)$$
$$= h((X_1 + N_2) \bmod A) - h(N_2 \bmod A)$$
$$\leq \log_2 A - h(N_2 \bmod A)$$

where the inequality is for all distributions with support restricted to [-A,A], and equality holds if X_1 is uniform.

Thus, choose X_1,X_2 uniform on [-A,A]. (will see later that this was not the best choice).

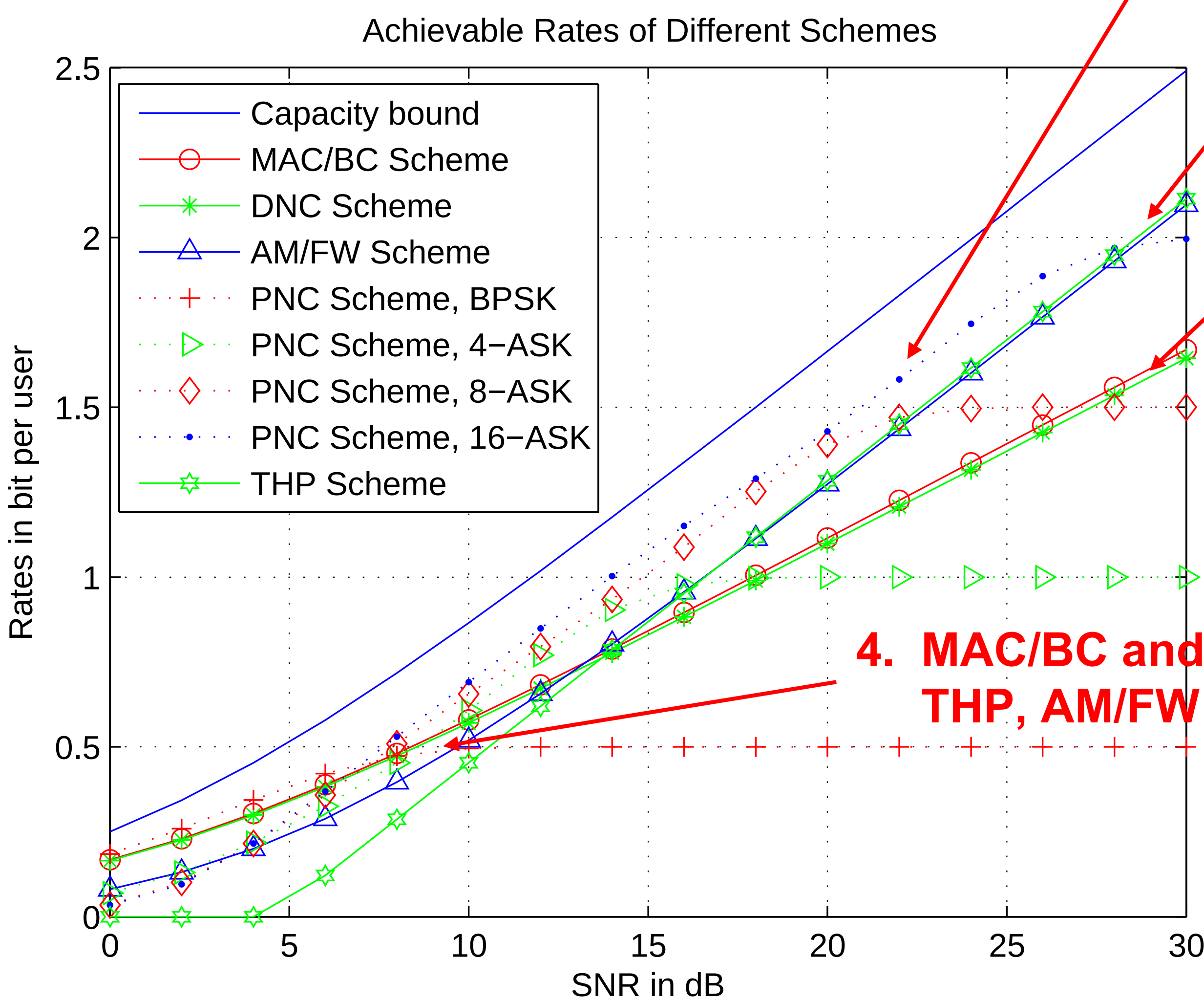
Numerical Results I

1. Envelope of PNC the best.

2. THP narrowly beats AM/FW at high SNR. (more on this later)

3. MAC/BC and DNC perform identically.

4. MAC/BC and DNC preferable to THP, AM/FW at low-to-moderate SNRs



Numerical Results: THP Observations

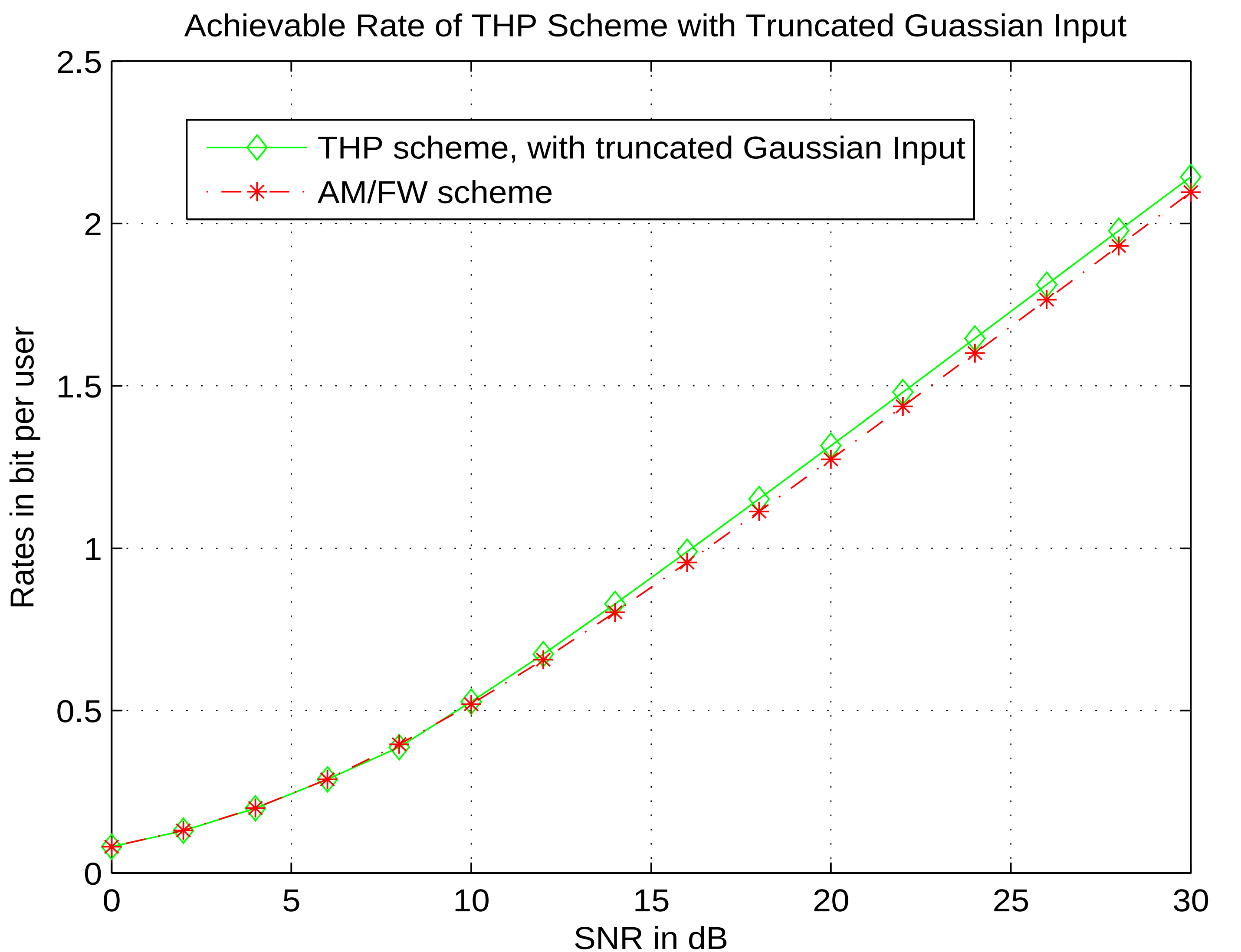
Observation: We expected a $5 \log_{10} 2 = 1.7$ dB gain for THP over AM/FW at high SNRs. Less than 0.2 dB observed.

Reason: Uniform distribution costs us 1.5 dB in *shaping gain* versus the Gaussian distribution.

Fix: Rather than maximizing mutual information under support constraint, take both power and support into consideration – a very difficult problem.

However, it is possible to guess reasonable distributions and then observe the results achievable – employ truncated Gaussian input distribution.

Numerical Results: Impact of Signal Shaping



Conclusions/Future Work

1. PNC scheme of Zhang et al nearly optimal on AWGN channels (no synchronization issues). Only loss of quantized channel-input/channel-output.

2. [Others] MAC/BC or standard network coding are preferable at low SNRs, AM/FW and THP-based preferable at high SNRs.

3. THP-based can slightly (0.4 dB) outperform AM/FW at high SNRs.

4. Extensions to fading channels is of supreme interest: some (AM/FW) extend naturally and only require analysis, others (PNC, THP-based) require both (non-trivial) design and analysis.