Bounds on the Gain of Network Coding and Broadcasting in Wireless Networks

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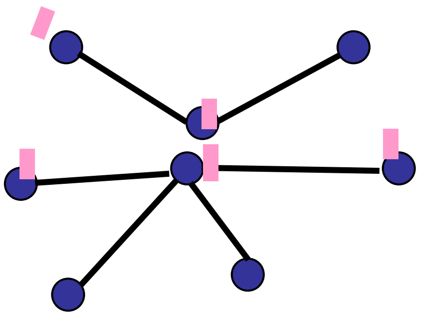
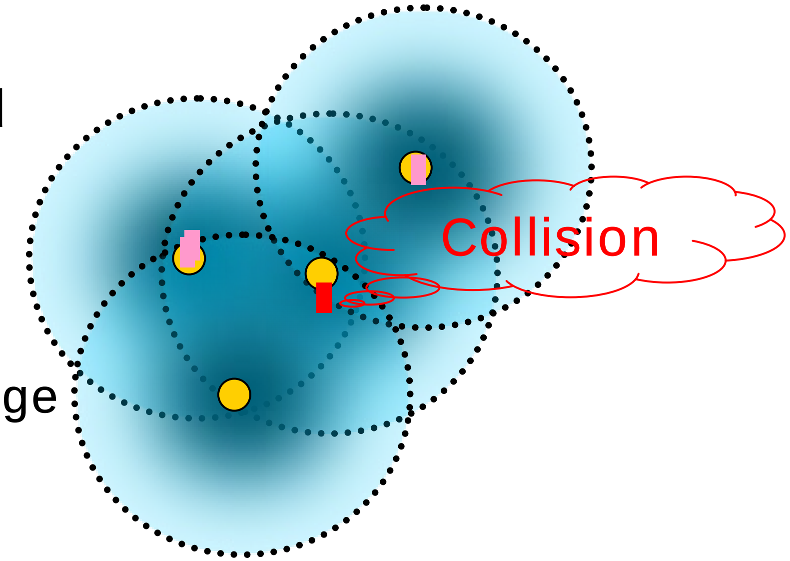
Wired vs. Wireless Communication

- Wired Communication
  - Physical signal constrained in Wired lines
    - High spatial usage
    - High infrastructure cost
    - Sender receiver position fixed



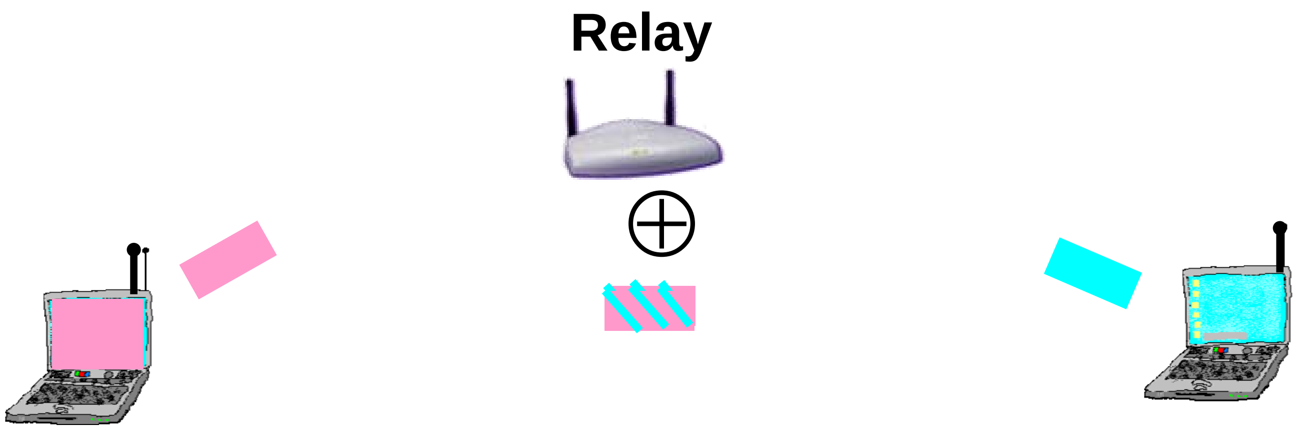
- Wireless Communication
  - Physical signal unconstrained in a range
    - Low infrastructure cost
    - Mobile ubiquitous comm.
    - Interference: low spatial usage



No Coding

4 Transmissions

Network Coding in Wireless: XOR in the Air



3 Transmissions

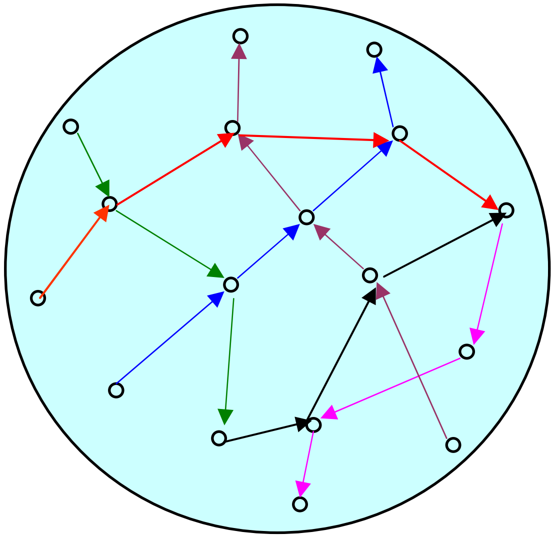
25% savings

33% throughput increases

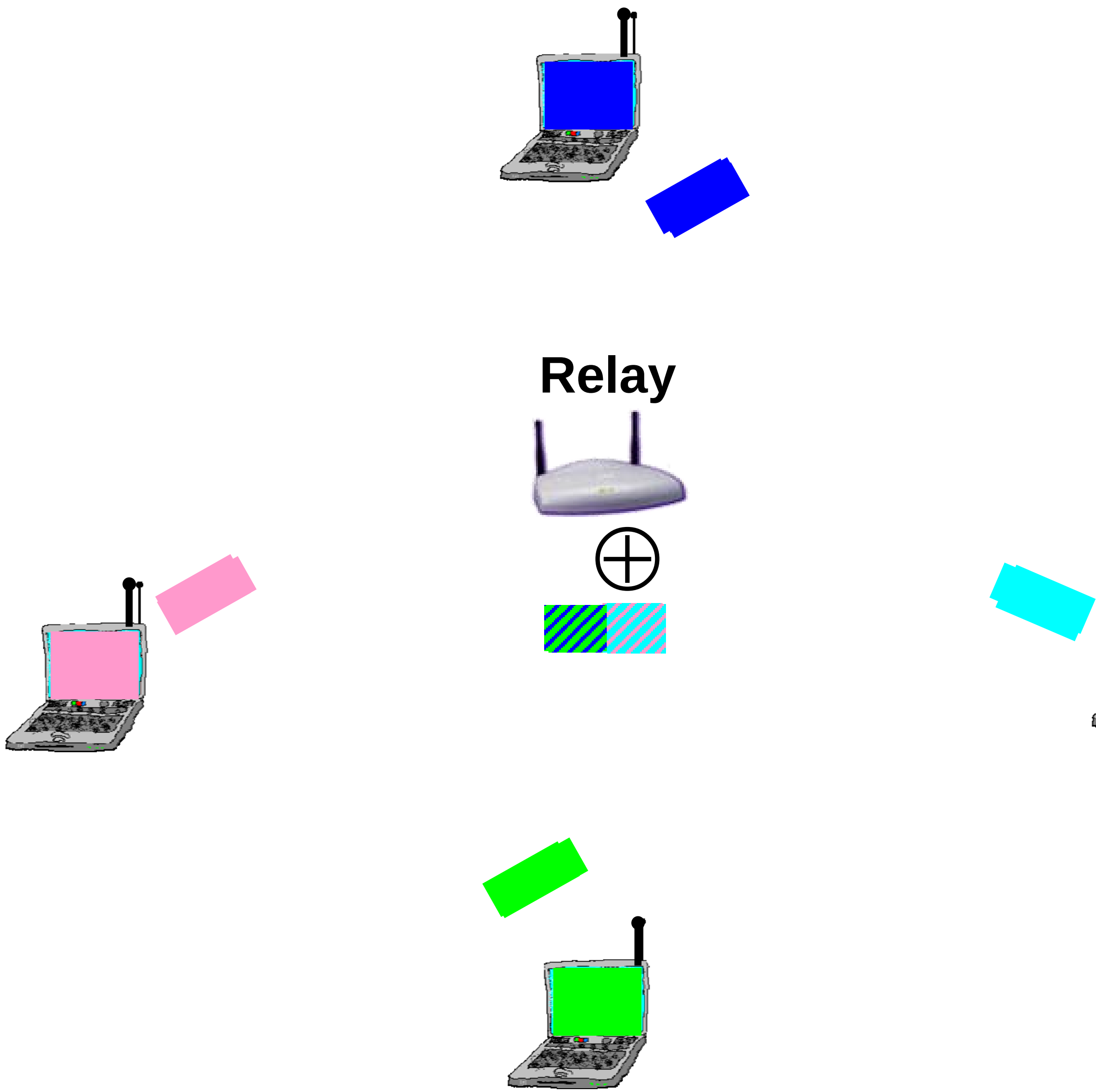
Wu et al Allerton 05, Katti et al Sigcomm06

Capacity of Random Wireless Ad Hoc Networks
[Gupta and Kumar99]

- n nodes, each send/receive W bits/sec
- Multipair unicast traffic in random ad hoc wireless network
- Per session capacity $\propto \frac{W}{\sqrt{n \log n}} \rightarrow 0, \text{ as } n \rightarrow \infty$



Network Coding + Wireless Broadcast



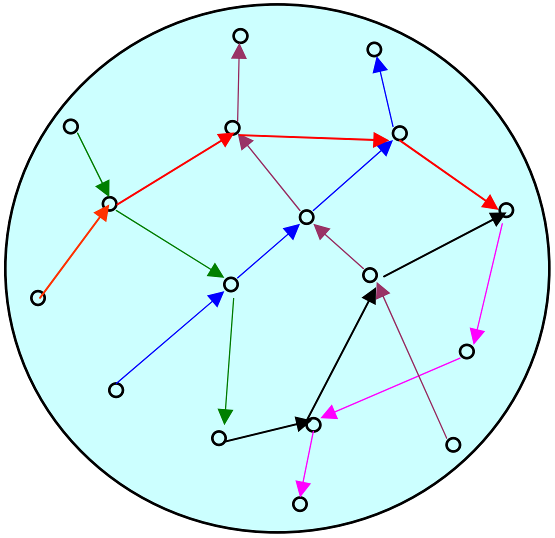
Network Coding + Wireless Broadcast

- Key insight:
  - To ensure connectivity communication radius $r(n) \propto \Theta\left(\sqrt{\log n / n}\right)$
  - NC + wireless broadcast potentially deliver information to $\log n$ neighbors in one transmission

How much gain can NC provide?

Model

- Gupta & Kumar model
  - *n* nodes randomly located in unit region, each can send/receive at W bits/sec
  - Traffic:
    - n random src-dest. pairs
    - Independent traffic content
    - Same rate

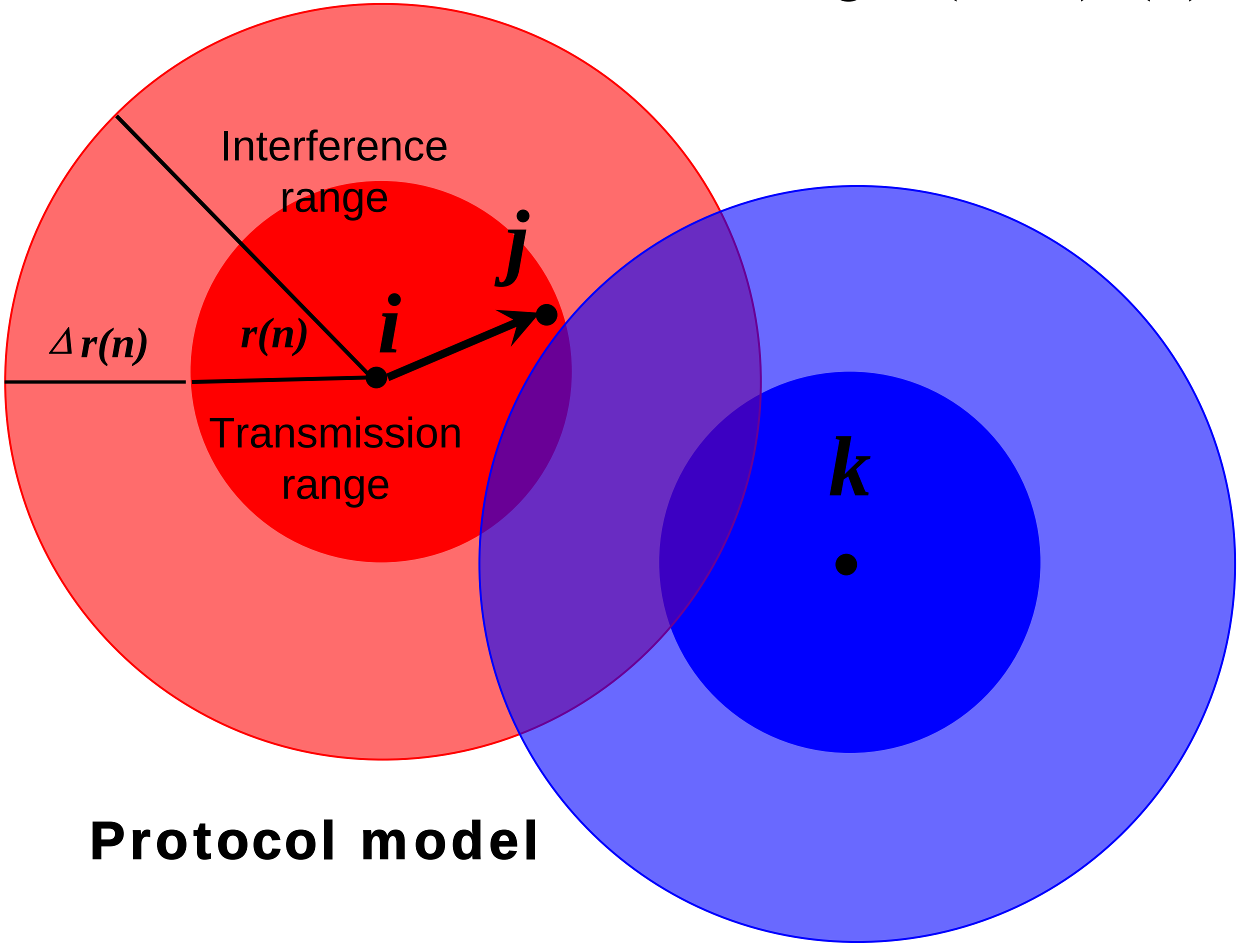


- Communication model
  - Protocol model
  - Physical model
- Difference:

network coding + wireless broadcast

Protocol Communication Model

- Successful transmission $i \rightarrow j$
  - j within i's transmission range $r(n) \propto \Theta\left(\sqrt{\log n / n}\right)$
  - j outside k's interference range $(1+\Delta)r(n)$



Protocol model

Physical Communication Model

- Transmission $i \rightarrow j$ success iff

  Signal to Interference & Noise ratio (SINR)

$$\frac{\dfrac{P}{r_{i,j}^{\alpha}}}{N + \sum_{k \neq i} \dfrac{P}{r_{k,j}^{\alpha}}} \geq \beta$$

  - P – transmission power
  - N – ambient noise
  - α – attenuation factor
  - β – threshold

Our Result

- For both Protocol and Physical model

$$\frac{\lambda_C(n)}{\lambda_F(n)} = \Theta(1)$$

$\lambda_F(n)$: flow throughput, without coding

$\lambda_C(n)$: coding throughput, enhanced by NC + wireless broadcast

Protocol Model – 1D

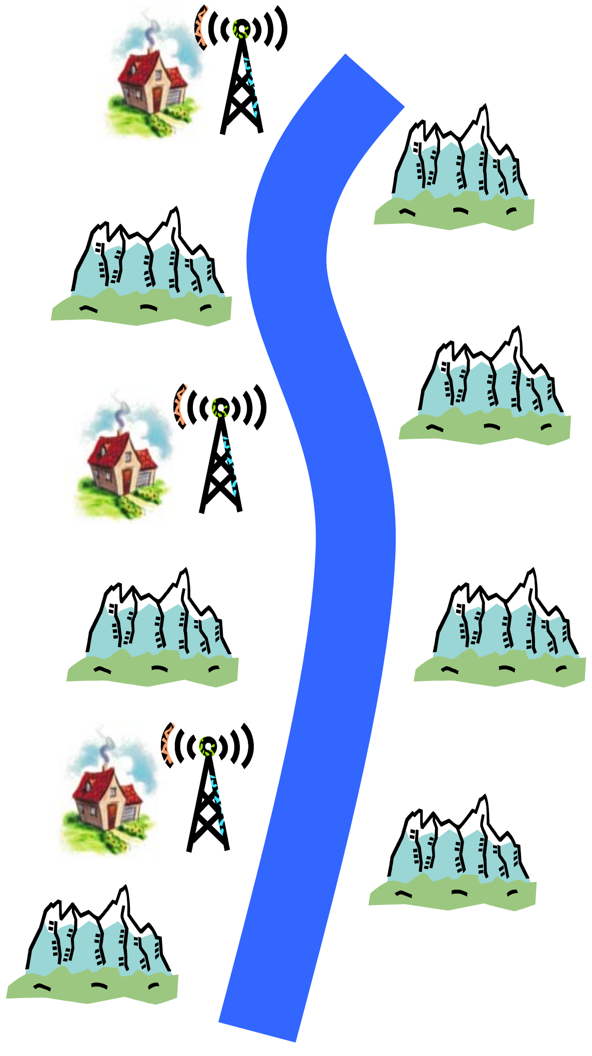
- 1D models valley
- Flow throughput

$$\lambda_F(n) = \frac{2W}{(1+\Delta)n}$$

- Coding throughput

$$\lambda_C(n) = \frac{2W}{(1+\Delta/2)n}$$

- Coding benefit factor
  - $\frac{\lambda_C(n)}{\lambda_F(n)} = \frac{1+\Delta}{1+\Delta/2} \in (1,2)$
  - 4/3 for 802.11 ($\Delta \approx 1$)



Protocol Model – 2D

- Flow throughput

$$\lambda_F(n) \geq \frac{W}{c_1\sqrt{\pi}(1+\Delta)nr(n)}, \quad c_1 = \max\{2, \sqrt{\Delta^2 + 2\Delta}\}$$

- Coding throughput $\lambda_C(n) \leq \frac{2W}{n}\left(\frac{1}{\Delta r(n)} + 1\right)$

- Coding benefit factor $\frac{\lambda_C(n)}{\lambda_F(n)} \leq 2c_1\sqrt{\pi}\frac{1+\Delta}{\Delta}$

  - <15 for 802.11 ($\Delta \approx 1$)

Physical Model

- Flow throughput lower bound

  1D: $\lambda_F(n) \geq \Theta\left(\frac{W}{n}\right)$ 2D: $\lambda_F(n) \geq \Theta\left(\frac{W}{\sqrt{n}}\right)$

- Coding throughput upper bound

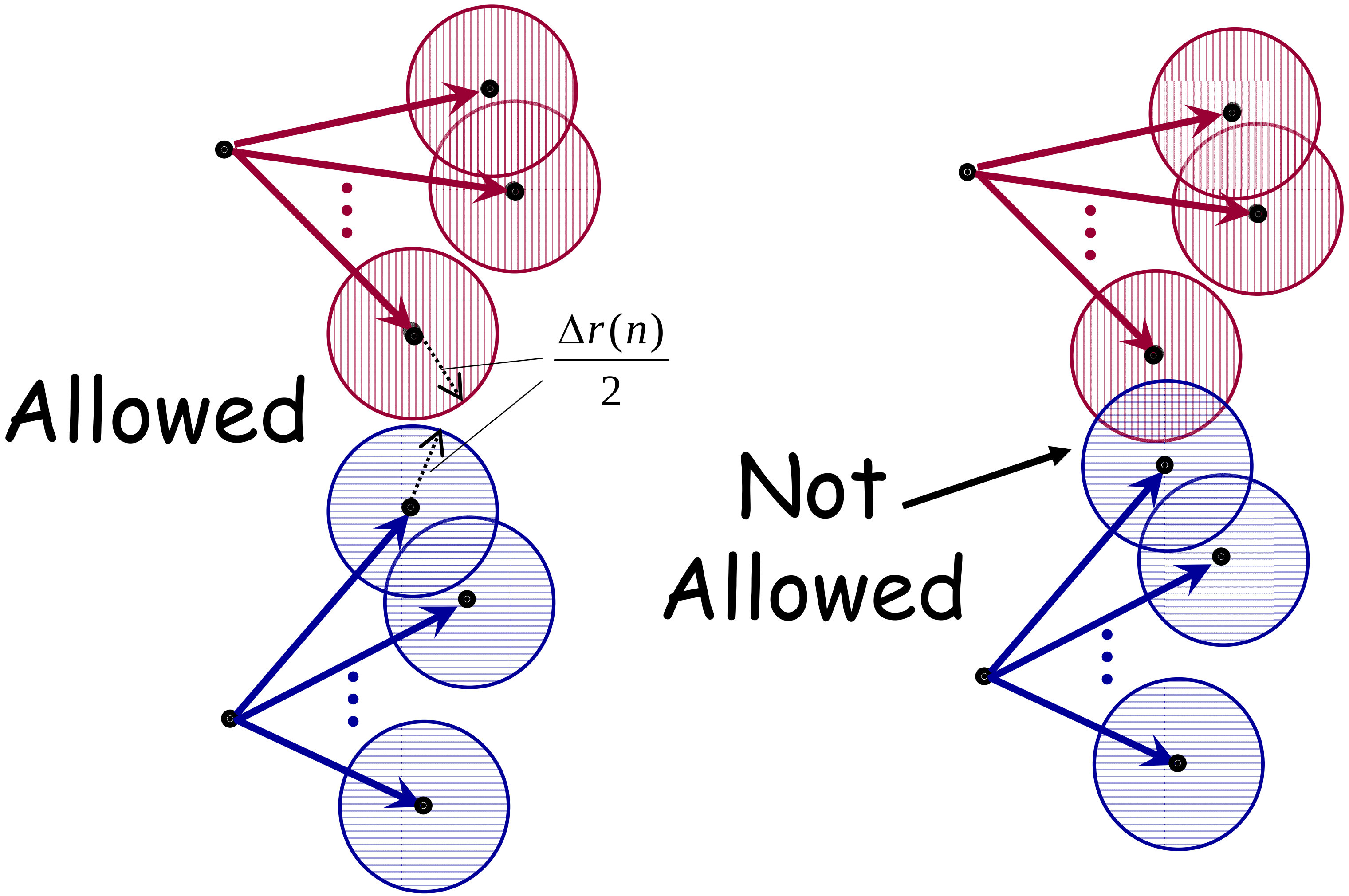
  1D: $\lambda_C(n) \leq \Theta\left(\frac{W}{n}\right)$ 2D: $\lambda_C(n) \leq \Theta\left(\frac{W}{\sqrt{n}}\right)$

- Coding benefit factor

$$\frac{\lambda_C(n)}{\lambda_F(n)} \leq \Theta(1)$$

Proof Techniques

- Geometric observation



Max Number Transmission - Cut Technique

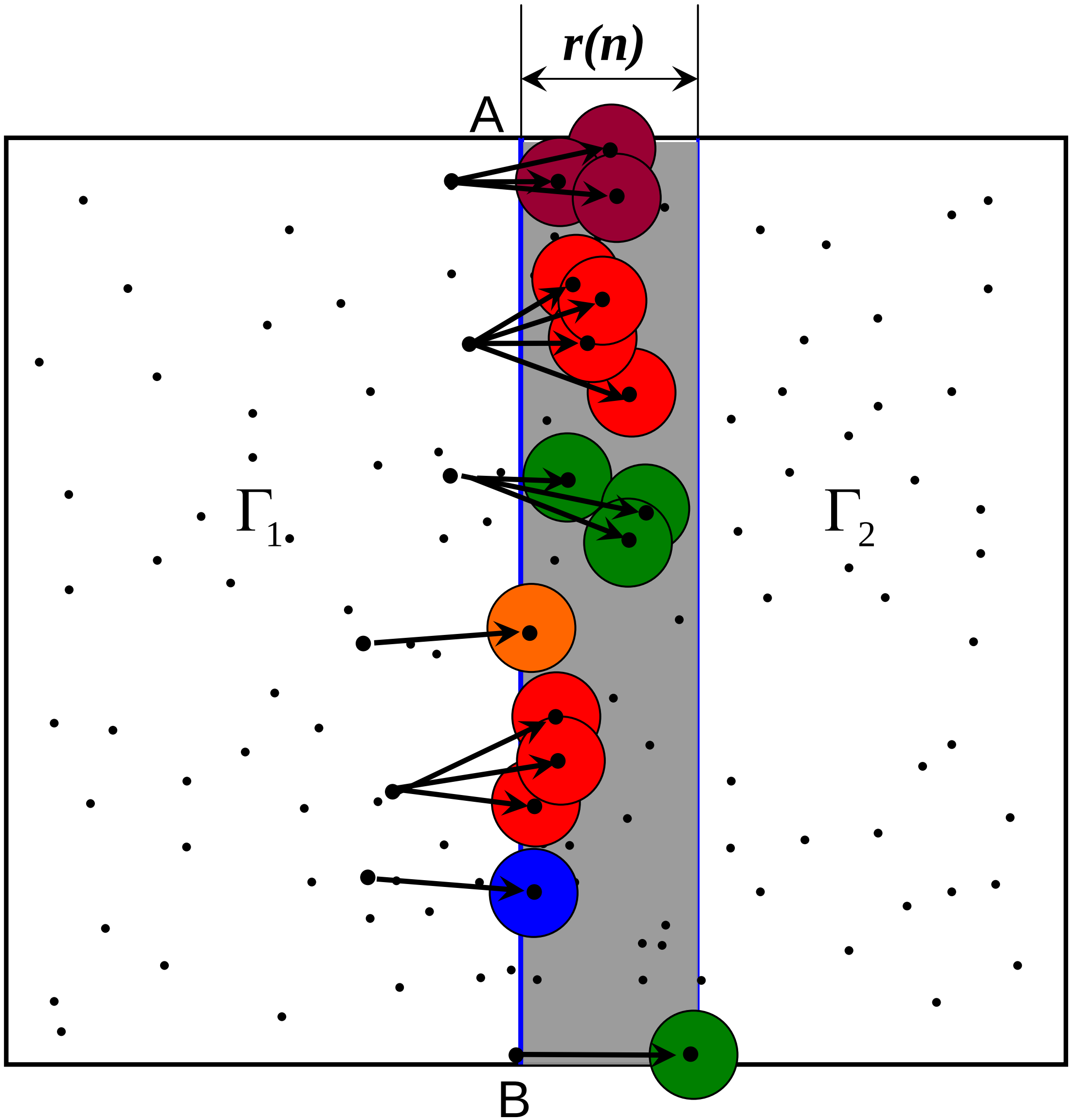
When Δ < 2

Each transmission consumes

$$\frac{1}{4}\pi\left(\frac{\Delta r(n)}{2}\right)^2 = \frac{\pi\Delta^2 r(n)^2}{16}$$

Maximum # transmissions

$$\leq \frac{16\,l}{\pi\Delta^2 r(n)}$$



Capacity Upper Bound

- Max transmissions across a cut: $\dfrac{c_{\Delta} l}{r(n)}$

  l – cut length

  $$c_{\Delta} = \max \left\{ \frac{16}{\pi \Delta^2}, \frac{\sqrt{3}}{\Delta} \right\}$$

- Cut Capacity $\leqslant$ max # transmission X W

- 2D cut capacity order $\Theta\left(\dfrac{\sqrt{n}}{\sqrt{\log n}} W \right)$

$\Rightarrow$ 2D throughput order $\Theta\left(\dfrac{W}{\sqrt{n \log n}} \right)$

Summary

Throughput gain		Performance		
		Flow Throughput $\lambda_F(n)$	Coding Throughput $\lambda_C(n)$	Benefit factor $\frac{\lambda_C(n)}{\lambda_F(n)}$
Protocol Model	1D	$\frac{2W}{(1+\Delta)n}$	$\frac{2W}{(1+\Delta/2)n}$	$=\frac{1+\Delta}{1+\Delta/2}$
	2D	$\geq \frac{W}{c_1\sqrt{\pi}(1+\Delta)nr(n)}$	$\leq \frac{2W}{n}\left(\frac{1}{\Delta r(n)}+1\right)$	$\leq 2c_1\sqrt{\pi}\frac{1+\Delta}{\Delta}$
Physical Model	1D	$\geq \Theta\left(\frac{W}{n}\right)$	$\leq \Theta\left(\frac{W}{n}\right)$	$=\Theta(1)$
	2D	$\geq \Theta\left(\frac{W}{\sqrt{n}}\right)$ [Franceschetti et al.]	$\leq \Theta\left(\frac{W}{\sqrt{n}}\right)$	$=\Theta(1)$

Energy saving: $\leqslant 3$

Backup Slides

Cut capacity for case of $\Delta \geq 2$