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Achievable Performance Improvements Provided by Cooperative Diversity

by

Steven Bohacek

University of Delaware

AGENDA

- Cooperative Relaying (Best Select)
- Exploitation of diversity
- Different topologies
- Correlation of Channel Gains
- Modeling of channel
- Performance Improvements and Results

Mobile Wireless Networks

- Mobile Wireless Networks
  - Variability of channels
- Mitigation of impact of time variability at the network layer
  - Pre-computed back up paths
  - Utilization of channel diversity
    - Single hop (multiple antennas)
    - Multiple hop network (link diversity)
      - Some links are better than others
- Diversity : Stochastic nature of channel
  - Channel gain modeled as independent random variable
    - Some channel provide better performance than others

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Motivation

- Diversity end to end performance gain
  - Up to 3000 times across a 5 hop network
- Reason
  - Dynamic range of physical layer like 802.11 exceeds 50 dB
- Challenge for Information Theorist
  - Achieve communication even when the channel is bad
- Option at network layer
  - Use other better channels

Problem Definition and Terminology

- Goal of diversity exploiting Routing Schemes
  - Utilize alternative routes that provide better performance
- Route Metric (for best path)
  - Maximum channel loss along the path
  - Or channel loss along the worst ling along the route
- Motivation to minimize the maximum channel loss
  - Loss rate falls to zero as SNR increases beyond threshold
    - Transmission power at each link adjusted to meet target SNR
  - The energy to deliver from source to destination
    - dominated by energy required to transmit along the worst link along the path
  - Throughput along a route given by bit rate along slowest link

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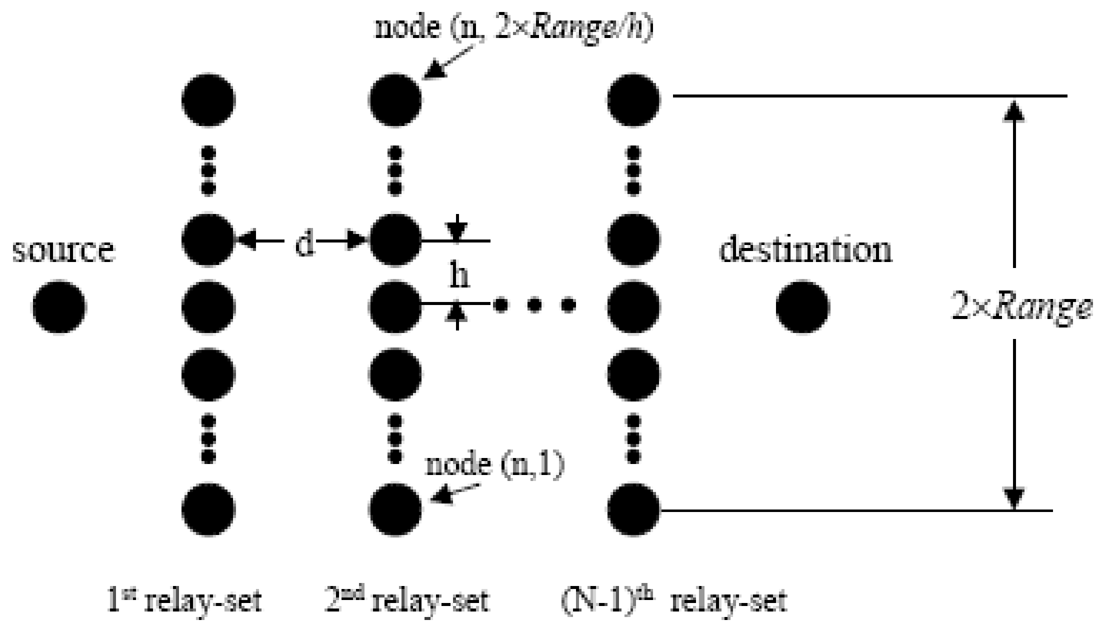
Topology for wireless network

Objective: To find best path from source to destination

Each hop must be between two nodes in adjacent relay sets



The distance between relay sets is d and space between adjacent nodes is h. Each relay set spans a vertical distance of 2XRange

Terminologies

- U (r): probability that the destination is occupied.
- Link *open*
  - if the channel loss is less than r
  - *closed* otherwise.
- Node *occupied*
  - if there is a sequence of open links from the source to the node.
- packet traversal:
  - from a node in the n-th to (n + 1)-th relay-set
  - the exact node within each relay-set that relays the packet can be adjusted
- Number of nodes in each relay set: M or 2X floor (Range/h)
- Node (n,i) : ith node in nth relay set
- Source node (0, floor(M/2)) ,Destination node (N, floor(M/2))

- Channel loss
  - Deterministic part
    - Distance between transmitter and receiver
  - Random part
    - Log normally distributed
- $q_{Threh}(|i\text{-}j|)$ = prob(link (n,i) and (n+1,j) is open)
- X is gaussian with mean 0 and deviation sigma
- $$q_{Thresh}\left(|i-j|\right) := P\left(\text{Thresh} > X + 2.7 \times 10\log_{10}\left(\sqrt{d^2 + h^2\left(i-j\right)^2}\right)\right) \quad (1)$$

Independent channel and dependent node occupancy

- Performance determination
  - Represent occupied nodes in relay-set as Markov chain
  - State of Markov chain
    - Vector telling which nodes are occupied/unoccupied

$$a \in \{0,1\}^M$$
$$M\_nodes\_in\_each\_relay\_set$$
$$a_i = 1\_i\_node\_is\_occupied$$
$$a_i = 0\_i\_node\_is\_unoccupied$$
$$State\_can\_be\_represented\_as\_an\_integer\_between$$
$$0\_to\_2^M - 1$$

Probability Transition Matrix for Markov Chain

- Equation 2.

$$Q_{Thresh}(A,B) := P(Moving\_from\_state\_A\_to\_B)$$

$$\prod_{\{i:b_i=1\}} (1 - \prod_{\{j:a_j=1\}} (1 - q_{Thresh}(|i-j|))) \times \prod_{\{i:b_i=0\}} (\prod_{\{j:a_j=1\}} (1 - q_{Thresh}(|i-j|)))$$

- Probability of node i not being occupied is the probability that each link from every occupied node in the previous relay set is closed which is given by

$$\prod_{\{i:b_i=1\}} (1 - \prod_{\{j:a_j=1\}} (1 - q_{Thresh}(|i-j|))$$

- Probability of node i being occupied is the probability that at least one of link from occupied node in the previous relay set is open which is given by

$$\prod_{\{i:b_i=0\}} (\prod_{\{j:a_j=1\}} (1 - q_{Thresh}(|i-j|))$$

- Probability distribution of the occupied nodes within the n-th relay-set:
  - Calculated using equation 2
  - Source node (0, floor(M/2))
  - V := probability distribution of the occupied nodes within the 0-th relay-set

$$V_A = 1\_if\_A = 2^{\lfloor M/2 \rfloor}$$

- The probability distribution of the set of occupied/unoccupied nodes n hops from the source is

$$VQ_{Thresh}^{n}$$

Where $Q_{Thresh} : 2^M \times 2^M \_matrix$

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- Destination : (N,floor(M/2))
- U: probability distribution in Nth relay set
- Probability (destination is occupied)=
  - found by summing the elements of U over all the states that have destination occupied.
- $$\tau(Thresh) = V \times Q_{Thresh}^{N} \times W$$

$$W_A = 1$$

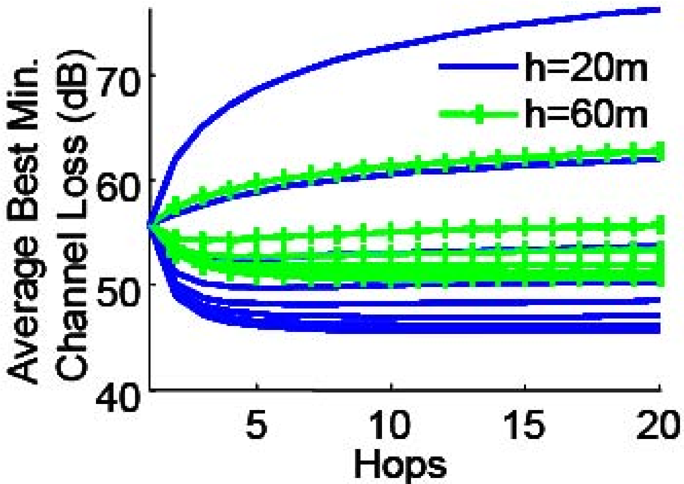
$$if$$

$$a_{\lfloor M/2 \rfloor} = 1$$

$$where$$

$$A = \sum_{i=1}^{M} a_i 2^i$$

Performance Analysis



Average maximum channel loss along the best path

Upper most curve when M=1 (only one node in each relay set)

As the size of relay set increases the maximum channel loss along best path decreases

Figure shows performance for relay set sizes of M=1,2,4,6,…,14

The best performance is for M=14 which corresponds to the lowest curve

Problem with Markov chain model

- In previous analysis, largest relay set size is M=14
  - 2^{14} in the state space
  - 2^{28} elements in the state transition matrix
- With the processors available, this is the largest topology
  - Whose performance could be evaluated in realistic time

Independent channel Independent Nodes

- Computation becomes easy
  - If occupancy of node is independent of whether other nodes in the same relay set are occupied
  - Independence assumption provides an upper bound on the performance
  - For large relay sets, independent assumption does not have significant impact on accuracy

Probability of occupancy under independence assumption

- $P_{n,Thresh}(j)$=Prob(j th node in n th relay set is occupied)

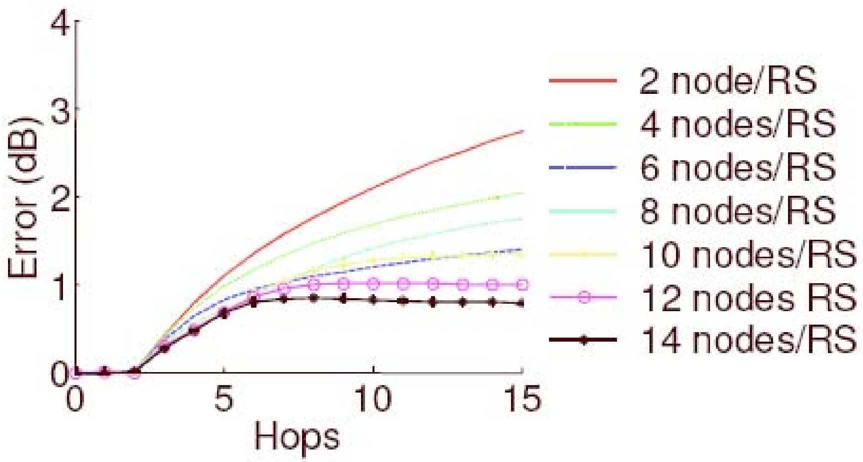
$$P_{n+1,Thresh}(i) = 1 - \prod_{j=1}^{M} (1 - q_{Thresh}(|i-j|) P_{n,Thresh}(j))$$

$P_{0,Thresh}(i)$ =1 for i = floor(M/2) and 0 otherwise

$P_{N,Thresh}$(floor(M/2)):=Prob(there exists a path from source to destination)

This can be easily computed

Approximate performance results



- Approximation yields results performance relationships quite close to exact one
- For large networks the error is less than 1 dB and converges as hops increases
- For small networks the error is larger, but then actual performance can easily be computed for such networks with small realy sets.
- Error is always positive implying that approximation is always larger than the actual value

Performance for some different topologies

$$P_{n+1,Thresh}(x,y) = \rho_n(u,v) \times \left(1 - \prod_{\substack{-d<2<u<d/2 \\ -\infty<v<\infty}} \left(1 - q_G(x-u, v-y)\, P_{n,Thresh}(u,v)\right)\right). \quad (6)$$

where

$$q_G(x-u, v-y) := P\left(\text{Thresh} > X + 2.7 \cdot 10 \log_{10}\left(\sqrt{(d+x-u)^2 + (v-y)^2}\right)\right),$$

with $X \sim N(0, 11)$.

Nth relay set consists of strip of strip of node centered along the line (nxd,0)+(x,y) where –d/2 <x<d/2 and –inf<y<inf

$\rho_n(x,y)$ is the probability that a node exists at (x,y)

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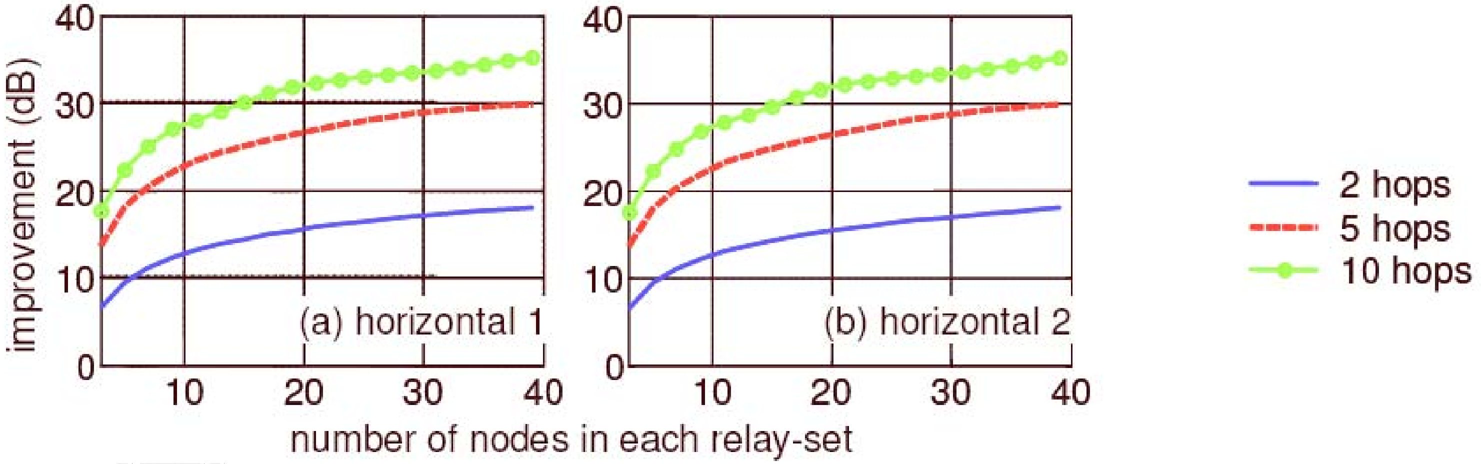
Results for different topologies



Topology description:

horizontal 1: nodes are uniformly spaced

horizontal 2: nodes are more clustered at the center

As the number of hops increases the improvement is more

As number of nodes in each relay set increases improvement is more

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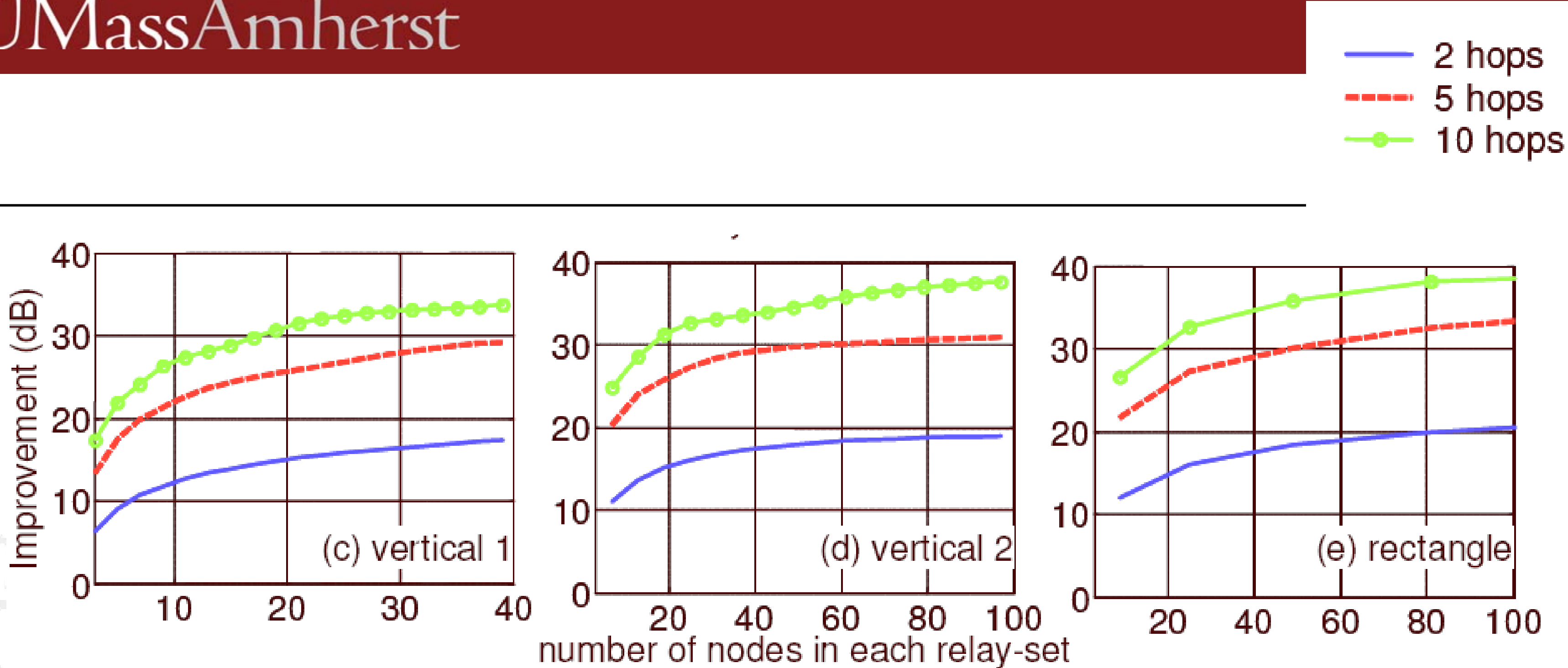


Topology description:

vertical 1: nodes are uniformly spaced

vertical 2: nodes are more clustered at the center

rectangle: nodes are more clustered at the center

As the number of hops increases the performance improves

As number of nodes in each relay set increases improvement is more

Performance as the number of nodes in each relay set are quite similar for different topologies

Correlated channel and independent node occupancy

- Performance improves when
  - Size of the relay set increases
  - Spacing between the nodes in the relay set decreases
- Both these trends are reasonable
  - As the size of relay set increases
    - More path is available
  - As spacing between the node sets decreases
    - Quality of path from single node to all the nodes in the next relay set increases
- What is the maximum performance that can be achieved
  - By increasing the number of nodes
  - Decreasing the distance between nodes in a relay set
- Major Difficulty
  - As h decreases the channels become correlated

Channel Correlation problem

- Major Difficulty
  - As h decreases the **channels become correlated**
  - Nodes (n,i) and (n,i+1) are closer together.
  - Channel between (n,i) and (n+1,j) and that between (n,i+1) and (n+1,j)
    - Pass through the same envirenment
    - Subject to same impairment
    - Have similar loss
- Model relay-sets as
  - Continuum of nodes
  - Channel as diffusion process or a random field
  - Use poisson clumping heuristic to approximate the performance

Two hop case

- Correlated channels subject to shadow fading have been reasonably studied
- Literature prescribes diffusion based model for channel loss
  - Stochastic part of channels modeled as Ornstein-Uhlenbeck process

$$\begin{aligned} dL_y^1 &= -\alpha L_y^1 dy + \sigma\sqrt{2\alpha}\, dB_y^1, \\ dL_y^2 &= -\alpha L_y^2 dy + \sigma\sqrt{2\alpha}\, dB_y^2, \end{aligned} \qquad (7)$$

Where L_1^y = shadow fading part of the channel loss (in dB) from src to node located at (d,y) in relay set

L^y_2 = shadow fading part of channel loss from node at (d,y) to destination

B^i_y brownian motion processes with B1 and B2 independent

Alpha = (1/10) per meter, sigma=11dB

Two hop case

$$L_y = N(0, \sigma)$$

$$E(L_y L_x) = \sigma^2 \exp(-\alpha(|y - x|))$$

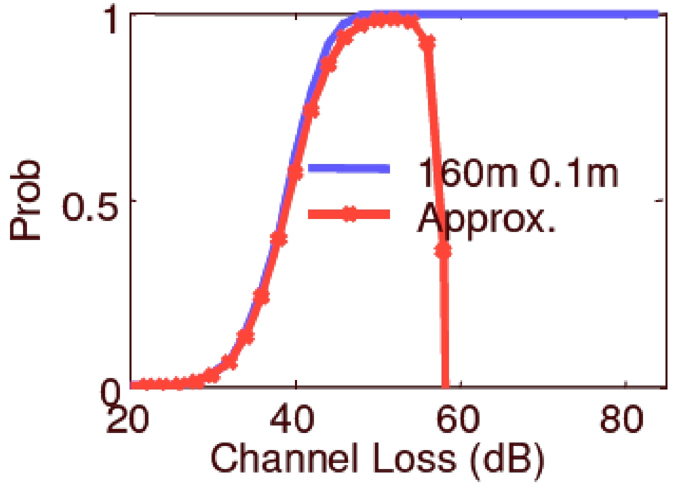
In the limit as node density goes to infinity, the probability that there exists a path from source to destination such that each link has channel loss less than *Thresh is given by*

$$\begin{aligned} U_\infty(Thresh, Range) :=& \\ & P\Big(\exists \text{ a } y \text{ such that } -Range < y < Range, \\ & L_y^1 + 2.7 \times 10 \log_{10}\left(\sqrt{d^2 + y^2}\right) < Thresh, \\ \text{and } & L_y^2 + 2.7 \times 10 \log_{10}\left(\sqrt{d^2 + y^2}\right) < Thresh\Big) \end{aligned} .$$

Such probability difficult to compute exactly

If Thresh is small, the probability of event occurring rare is approximated by Poisson clumping heuristic

Discrete channel simulation



- Fig shows the cumulative distribution of maximum channel loss along best path for h=0.1 for 2 hop network
- Relay sets were 160 m long and 100 m apart
- We can see that for channel loss of 40 to 60 dB , the probability is around 1
- For Channel loss less than 30 dB its almost 0
- And for channel loss of more than 60 dB is also zero

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Channels between two relay sets

- Performance between two relay sets
  - Correlation between channels is more complicated
- Consider channel between nodes
  - (nd,u) ((n+1)d,x)
  - (nd,v) ((n+1)d,y)
  - If x apprx = y & u appr=v then channel will be correlated
    - L(u,x) stochastic part of channel (nd,u) ((n+1)d,x),then

$$E(L(u,x)L(u,y))$$
$$=\sigma^2 \exp(-\alpha(|u-v|+|x-y|))$$

- It implies that L is a random field or L is a product Ornstein Uhlenbeck process

$$E(L(u,x)L(u,y))$$

$$\approx \sigma^2(1-\alpha|u-v|-\alpha|x-y|)$$

$$for$$

$$\alpha|u-v|+\alpha|x-y| small$$

The probability that there exists a channel between two relay sets with loss less than threshold

$$U_\infty^{RR}(Thresh, Range) := P\left(\exists\, x \text{ and } y \text{ such that } -Range < x, y < Range \text{ and } L(x,y) < Thresh\right) .$$

Poisson clumping heuristic canbe applied to product Ornstein-Uhlenbeck processes to approximate the probability

Performance over Multi hop network

- Technique 1: developed in Two hop case
- Technique 2: developed in channels between two relay states
- Approximate performance of multi hop network
  - Technique 1
    - Used for 1st and last hop
  - Technique 2
    - Used for intermediate hops

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- Thanks