Interlude: Transform Theory
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• Why transforms? - makes life easier
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- Why transforms? - makes life easier
- non-negative integer rvs - \( z \)-transform, probability generating function (pgf)
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- Why transforms? - makes life easier
- non-negative integer rvs - $z$-transform, probability generating function (pgf)
- nonnegative, real valued rvs - Laplace transform (LT)
$z$-transform
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Defn.
**z-transform**

**Defn.** Have rv $X$ that takes nonnegative integer values

$p_k = P(X = k), \ k = 0, 1, \ldots$
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$$G_X(z) \equiv E[z^X] = \sum_{k=0}^{\infty} p_k z^k$$
$z$-transform: examples
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G_X(z) = \sum_{k=0}^{\infty} (1 - p)p^k z^k = \frac{1 - p}{1 - pz},
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for $pz < 1$
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$$= e^{-\lambda} \sum_{k=0}^{\infty} (\lambda z)^k /k!,$$
\[ = e^{-\lambda(1-z)} \]
Benefits
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moments:

\[ \frac{dG_X(z)}{dz} = \sum_{k=1}^{\infty} k p_k z^{k-1} \]
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which translates to

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similarly

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\[ G_U(z) = G_X(z)G_Y(z) \]
Solution of $\mathcal{M}/\mathcal{M}/1$ Queue Using Transforms
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Balance equations:
Solution of $M/M/1$ Queue Using Transforms

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$$(\lambda + \mu)\pi_i = \lambda\pi_{i-1} + \mu\pi_{i+1}, \quad i = 1, \ldots$$
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Multiplying by $z^i$, using $\rho = \lambda/\mu$, and summing over $i$
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$$(1 + \rho) \sum_{i=0}^{\infty} \pi_i z^i =$$
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$$(1 + \rho) G_N(z) =$$
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\[(1 + \rho)G_N(z) = \rho z G_N(z) + z^{-1}(G_N(z) - \pi_0) + \pi_0\]

Multiplying by $z$ and rearranging yields
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Multiplying by $z$ and rearranging yields

\[(\rho z^2 - (1 + \rho)z + 1)G_N(z) = \]
Solution of $\mathcal{M}/\mathcal{M}/1$ Queue Using Transforms

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Now,

$$\rho z^2 - (1 + \rho)z + 1 = (1 - z)(1 - \rho z)$$

which substituted into the above expression yields

$$G_N(z) = \pi_0/(1 - \rho z),$$
\begin{equation}
1 - \rho = \frac{1 - \rho}{1 - \rho z}
\end{equation}
Inversion of z-transforms
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We have looked at

\[ \{p_0, p_1, \ldots \} \rightarrow G_X(z) \]
Inversion of z-transforms

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\[ \{p_0, p_1, \ldots \} \rightarrow G_X(z) \]

How about

\[ G_X(z) \xrightarrow{\text{invert}} \{p_0, p_1, \ldots \} \]
The definition is:
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\[ p_n = \frac{1}{n!} \frac{d^n G_X(z)}{dz^n} \bigg|_{z=0} \]
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Not always easy to compute and not focus of this lecture.

Note that \( p_0 = G_X(0) \) (and \( G_X(1) = 1 \))
Laplace Transform
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Defn.
**Laplace Transform**

**Defn.** Have nonnegative, real valued rv $X$ with density function $f_X(x) \ (f_X(x) = 0, \ x < 0) \ k = 0, 1, \ldots$
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$$F^*_X(s) \equiv E[e^{-sX}] =$$
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**Example**
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**Example**

- \( X \) is exponential rv, \( f_X(x) = \lambda e^{-\lambda x} \)
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Moments:
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\[
E[X] = - \left. \frac{d}{ds} F_X^*(s) \right|_{s=0}
\]

\[
E[X^i] = (-1)^i \left. \frac{d}{ds} F_X^*(s) \right|_{s=0}
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Properties of Laplace Transform
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- convolution:
Properties of Laplace Transform

- **convolution:** if $X_1, X_2, \ldots, X_n$ are independent, nonnegative rvs with transforms $F_{X_1}(s), \ldots, F_{X_n}(s)$,
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$$F_Y(s) =$$
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F_Y(s) = F_{X_1}^*(s)
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Example, if $Y$ is $n$-th Erlang,
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F^*(s)
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Example, if $Y$ is $n$-th Erlang, then

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Properties of Laplace Transform

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Example, if \( Y \) is \( n \)-th Erlang, then

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Let $X_1, X_2, \ldots$ be iid nonnegative rvs with LT $F_X^*(s)$. 
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what is $F_Y^*(s)$?
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what is $F_Y^*(s)$?

$$F_Y^*(s) = G_N(F_X^*(s))$$